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CRACK-ARREST AT A BIMATERIAL INTERFACE

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Abstract—A study of the behavior of a transverse crack propagating at right angles through the interface of a bimaterial plate was undertaken by means of the method of high-speed photography along with the optical method of transmitted caustics. The investigation was mainly concentrated on the changes of the stress intensity factor at the tip of a moving crack when reaching the bimaterial interface. The investigation was also extended to duplex plates consisting of the same material in both phases of the duplex.

The results have shown that the process of crack arrest at the interfaces of duplex specimens depends mainly on the material discontinuity that this interface region presents on the process of crack propagation and not on the abrupt change of the mechanical properties of the materials of the phases.

Therefore, the dynamic problem of a crack reaching an interface can in no way be appropriately approached by means of only the results of the static analysis.

By studying the dynamic process of crack arrest at an interface we have formulated a rule for the experimental determination of the stress intensity factor during the whole process of crack arrest.

Finally and in relation with the above experimental evidence, the dependence between crack propagation and stress wave emission from the tip of a moving crack was also discussed.

1. INTRODUCTION

Among the papers dealing with the plane problem of crack propagation in two-dimensional plates there are only a few of them, which investigate problems of crack propagation through material interfaces or other definite material discontinuities (holes, transverse cracks, etc.). Nevertheless, a common result of all these—generally experimental—investigations is that the fast running crack reaches a state of arrest when it is arriving at a discontinuity, to initiate again, later on, with a different, in general, propagation velocity and stress field around the crack tip.

The authors of the present paper have already in the past suggested—investigating the fracture process of bimaterial plates—that the mechanism of crack arrest under such conditions is definitely related to the process of stress-wave emission from the crack tip during fracture[1, 2].

The aim of the present paper is to investigate, by means of an extensive experimental evidence, the process of crack arrest and transfer of the stress–strain field at a bimaterial interface by concentrating the experimental study at the close vicinity of the crack arrest area.

A effort was also made to illustrate the existing correlation between the crack arrest at an interface and the stress-wave emission from the tip of the propagating crack.

2. STRESS WAVE EMISSION DUE TO FRACTURE AND ITS INFLUENCE ON CRACK PROPAGATION

It is well known[3] that fracture of an elastic medium is accompanied by emission of stress waves from the tip of the propagating crack.

The mechanism of stress wave emission is associated with the process of removal of stresses at the crack-tip—as the crack propagates—from a stress level equal to the tensile strength of the material down to zero[4].

Experimental approaches of the problem have shown[5, 6] that, at both edges of a growing crack, all three types of stress waves are excited (longitudinal, transversal, Rayleigh) and that the characteristics of the Rayleigh stress waves are related to the stored elastic energy at the crack tip before fracture[7, 8].

On the other hand, some recent experimental investigations on the propagation of simple longitudinal or transversal pulses in both half-planes and finite plates have shown[9, 10] that a complicated wave pattern is soon established in the specimens as the initial longitudinal pulses decay by being split up in arrays of longitudinal and transverse pulses.

The above statement is closely related to the case of a continuous stress-wave emission from the tip of a moving crack in a plane specimen.

It becomes, thus, clear that the relation between fracture stress-waves and crack propagation can be in principle approached by means of average effects; that is, by investigating the overall characteristics of the stress-wave field emanating from the crack tip and the influence it can have on the fracture process itself.

Until now there are only a few investigations which deal with these phenomena: Bodner[11] explained the rather complicated fracture process of notched specimens of rectangular cross section by bending and the unpredictable variations of the K_I-values by means of the interference of the fracture stress waves—reflected at the free ends of the specimens—with the singular stress field. Kobayashi *et al.*[12] gave a similar explanation for the fluctuation of the K_I-value at the tip of crack approaching a hole. Kalthoff *et al.*[13] correlated again the fluctuation of the dynamic K_I stress intensity factor around the static K_I-value after crack arrest in DCB specimens to the influence of the fracture stress waves reflected at the finite boundaries of the specimen.

These works refer to the role that the stress waves, already emanated from the crack tip, can have on the further process of crack propagation. They leave, though, out of consideration the strong correlation between the moving crack tip singular field itself and the mechanism of stress-wave emanation. This correlation, according to our opinion, may be to a certain extent revealed by studying the process of a crack passing through a definite material discontinuity and especially a bimaterial interface.

Investigating the fracture of duplex epoxy-resin specimens under tensile loads[1, 2] the authors have in the past concluded that the inability of a continuous stress-wave propagation ahead of the crack tip, when the crack has arrived at a very close distance from the interface, results in a disturbance, or even in a destruction of the singular stress field at the crack tip, which forces the crack to arrest. The external load, when it is still acting on the specimen during the crack arrest time, has to create a new singular stress field, in order to cause a crack reinitiation.

3. THE OPTICAL METHOD OF CAUSTICS APPLIED TO CRACK PROPAGATION

The study of the stress intensity factors at the tip of the propagating crack and the crack propagation velocities was based upon the optical method of transmitted caustics.

According to this method a convergent or divergent light beam impinges on the specimen at the close vicinity of the crack tip and the transmitted rays are received in a reference plane, parallel to the plane of the specimen. The stress concentration at the vicinity of the crack tip results in a reduction of the thickness of the specimen at this region, due to the Poisson's ratio effect and to a change of the refractive index of the material at this area. The reflected rays from the neighborhood of the crack tip are concentrated along a strongly illuminated curve (caustic) on the reference plane, while the area enveloped by the caustic becomes a shadowed area.

From the geometrical characteristics of the caustic we can calculate, with high accuracy, the stress intensity factor at the tip of the moving crack. On the other hand, from the position of the caustic, we can accurately locate the position of the crack tip at any instant of time.

For a detailed description of the optical method of caustics see Ref. [14].

The method of caustics was selected for this study because it allows the closest approximation to the stress singularity as compared with all the other experimental methods, an advantage which is indispensable in our case when one wants to study the eventual stress redistributions at the crack tip during the period of crack arrest.

Moreover, the method of caustics is independent of the constant part of the stress field near the singularity since it is only interested in the variation of the sum of the principal stresses at the area where it applies, the expressions of the caustic depending only on the first and second derivatives of the complex stress function $\Phi(z)$.

The residual shrinkage stresses at the interface of the duplex expoy-resin specimens of the tests could be considerably reduced by a special treatment and generally they have a much lower magnitude than the singular stresses at the crack tip, but they have not completely vanished. However, this residual-stress field at the interface may be supposed as constant since its variation along the interface is rather smooth. Thus, the shrinkage stresses developed at the interface have no significant influence on the shape and dimensions of the caustic approaching the interface, which is once more an important advantage of the method of caustics over both photoelasticity and interferometry.

4. EXPERIMENTAL ARRANGEMENT

For the present experimental investigation we used a series of bimaterial epoxy-resin specimens with a length of 0.3 m, a width of 0.10 m and a thickness of 0.002 m. The specimens were composed of two full length phases of equal width (0.3 m × 0.05 m) bonded together along their common interface.

The mechanical properties of the two phases of the specimen were varied by adding different amounts of plasticizer (Thiokol-LP3) to the pure epoxy prepolymer (Epikote 828, Shell Co.). The two phases were bonded together without any adhesive joint. This was achieved by casting the material of phase II along the longitudinal boundary of the phase I-strip, which was already prepared and appropriately placed in the mould.

After consolidation of phase II the composite specimen obtained was thermally treated, so that a complete polymerization of the plates was achieved and, furthermore, a considerable reduction of the remaining shrinkage stresses was assured[2].

Phase I always contained an initial transverse slit of constant length $a_0 = 0.02$ m, which had a maximum distance between its adjacent lips less than 0.0003 m. This slit was introduced to initiate the crack at the same position of phase I.

Twenty different specimens were prepared by using five different types of compositions in all possible combinations with one another as phase I or phase II. These different compositions contained amounts of plasticizer increasing by 10% in each specimen; from zero up to 40%. Table 1 gives the static mechanical and optical properties of the various plasticized polymers used in our experiments[15].

Ten additional specimens were also prepared of the same dimensions as the previous ones, which had their phases I and II made of identical materials and properly cemented to each other phase. The amounts of plasticizer used in these specimens were again increased for each duplex specimen from zero up to 40% by amounts of 10%.

Moreover, another series of experiments was executed with ten plexiglas specimens, each of which was composed out of two strips of equal dimensions ($0.30 \times 0.05 \times 0.004\ \mathrm{m}^3$) bonded together along their longitudinal boundaries by means of a plexiglas powder-chlorophorm adhesive suspension. Phase I contained again a transverse initial slit of 0.020 m. The thickness of the adhesive film was of the order of some microns and it did not influence the behavior of each duplex specimen.

The dynamic load applied to the specimens was of the type of a falling weight with a constant load rate, creating a stress rate $\dot{\sigma} = 1 \times 10^8\ \mathrm{N/m^2\,s}$. The load was recorded with the help of an electrical dynamometric unit connected in series with the specimen, the signal of which was recorded by means of a storage oscilloscope. For the study of the crack propagation process we used a Cranz–Schardin high-speed camera, disposing 24 sparks with a maximum frequency of 10^6 frames per sec. With the help of an external generator the sparks can be set in

Table 1. Mechanical and optical constants of pure and plasticized epoxy polymers

Plasticizer C%	Young's Modulus $E[\mathrm{N/cm^2}]$	Poisson's ratio ν	Stress optical coefficient $C_t[\mathrm{m^2N^{-1}.10^{-2}}]$	Coefficients of optical anisotropy ε
0	324.000	0.338	17.03	0.1401
10	316.000	0.340	20.03	0.1161
20	292.000	0.344	20.79	0.1080
30	245.000	0.358	21.66	0.0983
40	179.000	0.430	30.78	0.0869

work with any desired frequency. The regulation of the spark frequency can also be made so that spark groups can have different frequencies, a fact which enabled us to study in more detail the fracture process at certain positions of the specimen. The synchronization of the fracture process with the high-speed camera was achieved by means of a silver-contact circuit, which triggered the sparks with the initiation of the crack propagation.

Figure 1 shows the block diagram of the experimental set-up.

The optical part of the experimental arrangement is shown in Fig. 2. The light beam from each spark reflects on a spherical mirror of a great reflective ability, with a diameter of 0.050 m and a focal distance of 7.0 m and after passing through the specimen it was focussed on each respective lens of the camera.

5. RESULTS

As mentioned before[1], the crack after propagating with a constant maximum velocity in phase I of the bimaterial specimen, decelerates abruptly, within a very small distance from the interface, and finally completely stops when it reaches the interface.

As the crack approaches very close to the interface and especially after the time needed for a complete crack arrest elapsed, a progressive disappearance of the caustic in phase I takes place, whereas the remaining part of the shape of the caustic is left unaffected, preserving its initial shape and dimensions of the generalized epicycloid. Furthermore, during this progressive disappearance of the caustic in phase I there are no signs that a new caustic is in the process of appearance and development in phase II.

Subsequently, and after the caustic in phase I is completely sunk in the interface, while the crack remains still at the arrest condition, a new caustic is continually emerging and rising progressively in phase II. The new caustic has initially the form of an arc of a circle, to emerge in a few microseconds as a full semicircle with its diameter on the interface.

Figure 3 presents a series of photographs showing crack propagation in a bimaterial plate

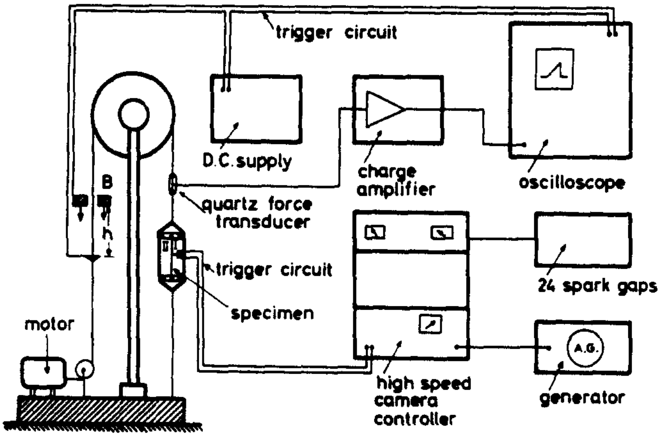


Fig. 1. Block diagram of the experimental set-up.

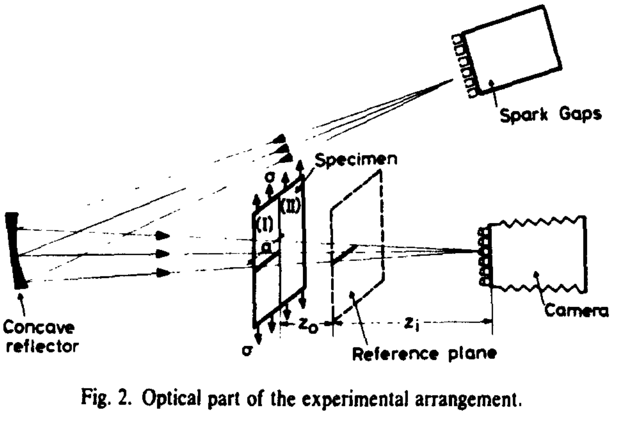
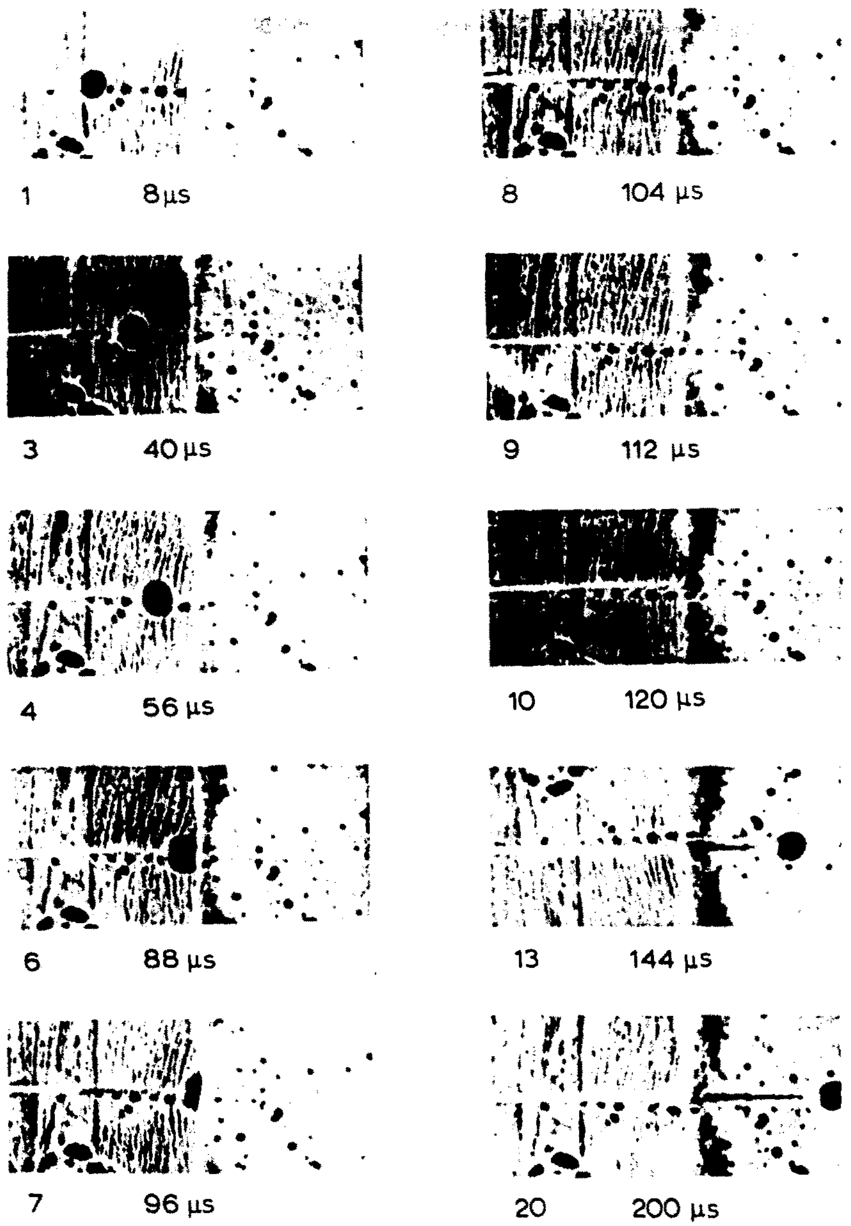


Fig. 2. Optical part of the experimental arrangement.


1 8 μs

8 104 μs

3 40 μs

9 112 μs

4 56 μs

10 120 μs

6 88 μs

13 144 μs

7 96 μs

20 200 μs

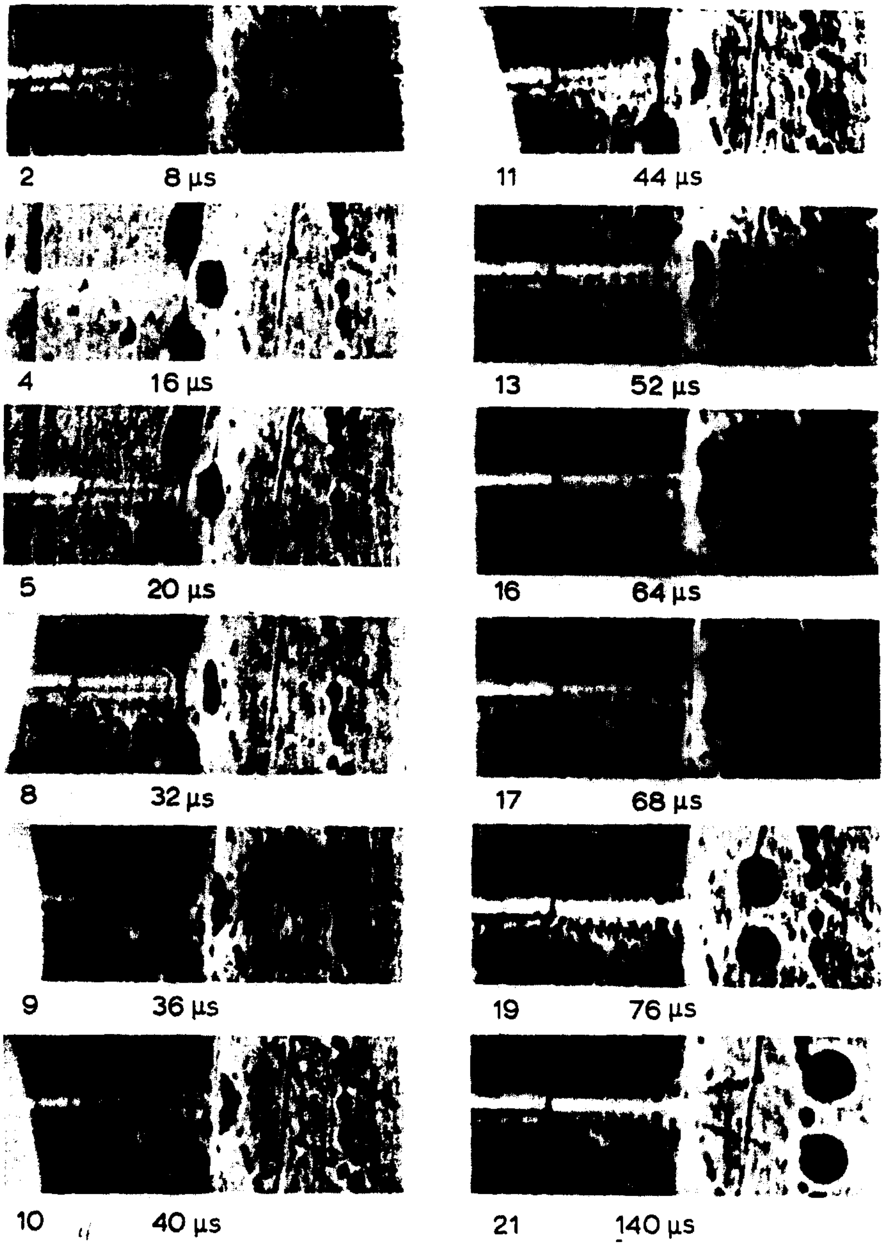
Fig. 3. Series of photographs showing crack propagation in a bimaterial specimen (phase I 30%, phase II 10% plasticized).

having a 30% plasticized epoxy resin in phase I and a 10% plasticized in phase II (30–10 specimen). Notice that the caustic does not appear in the second phase of the specimen during the whole period of crack deceleration and the first period of crack arrest although the crack tip, lying approximately at the center of the caustic, practically "touches" the interface.

Our experiments on the epoxy-resin bimaterial specimens have shown that all the above described qualitative characteristics of the crack arrest process remained unchanged for all possible material compositions of the bimaterial specimens. This result implies that the crack arrest process is, to a great extent, independent of the mechanical properties of the phases and is mainly related to the existence of the bimaterial interface.

In order to verify the above hypothesis we have studied afterwards the fracture procedure in duplex plates with identical epoxy compositions for both phases. Figure 4 presents the crack-propagation in a 20–20% epoxy polymer duplex specimen. Frames 5–11 show the period of crack arrest at the interface, since more than half of the caustic has been submerged at the interface.

Frames 4–8 show the submergence of the caustic at the interface. Notice the significant



2 8 μs
11 44 μs
4 16 μs
13 52 μs
5 20 μs
16 64 μs
8 32 μs
17 68 μs
9 36 μs
19 76 μs
10 40 μs
21 140 μs

Fig. 4. Series of photographs showing crack propagation and bifurcation at the interface of a duplex specimen with identical phase I and phase II (20% plasticizer).

deformation of the interface as it absorbs the kinetic energy of the crack and the strain energy stored at the crack tip.

Frames 9–11 illustrate the second period of the crack arrest interval. The specimen was fractured in a relatively high strain rate, which resulted in a considerable deformation of the interface (frames 4, 5 and 8 see also frame 8 of Fig. 3) and a crack bifurcation after crack arrest is developing (see [2]).

In frames 13–21 the development of the bifrucation process is presented which started after the semicircular caustic was built up in phase II during the last period of crack arrest.

A similar process was studied in Ref. [1]: The propagating crack, after being arrested at the interface, did not enter phase II, but moved at first along the interface, in two branches on both sides of the arrested transverse crack. Again, as the crack reached the interface (see Ref. [1] Fig. 10), the caustic was submerged in it and gradually disappeared from phase I, to emerge in a few microseconds in phase II as a semicircular caustic in touch with the interface. The disruption of the interface was connected with the split up of this arrested caustic form in phase

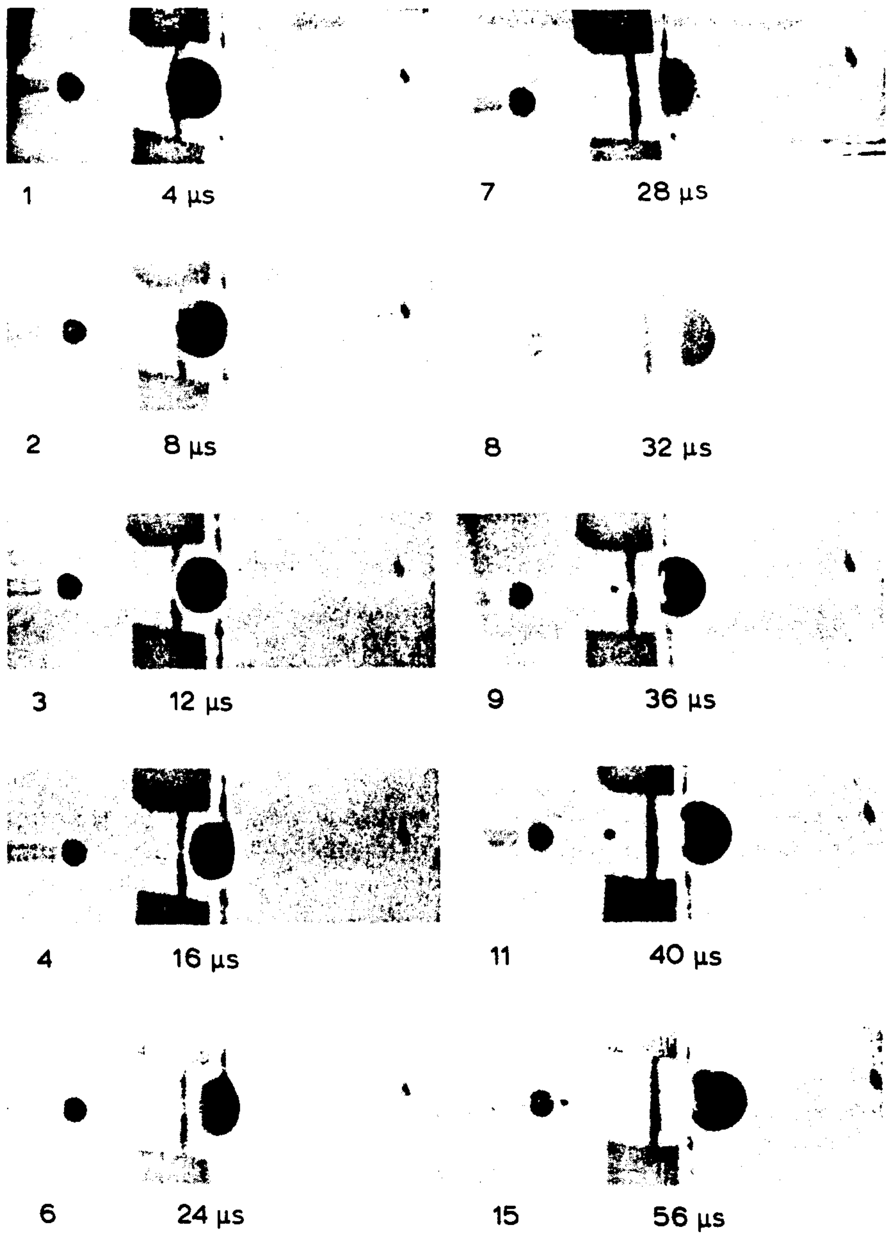


Fig. 5. Series of photographs showing crack propagation through the interface of a plexiglas duplex specimen.

II into two, smaller, semicircular caustics which moved in opposite directions along the interface, to enter finally, as full caustics, in phase II of the specimen. (A more detailed study of this phenomenon of interface disruption is under development.)

The experiments on duplex identical-material specimens were extended also to plexiglas specimens composed out of two strips with equal dimensions and cemented together along their common interface.

Figure 5 illustrates the fracture process of such a duplex plexiglas specimen. The crack arrest at the interface and the subsequent submergence—emergence process of the caustic, the deformation of the interface (frames 3, 4, 6) and the emission of fracture stress waves immediately after crack reinitiation (frame 8) are apparent in this figure. Notice, also the much larger, than in the case of an epoxy resin specimen, deformation of the interface, to the right as the crack arrives (frame 6) and thereafter to the left as the crack leaves (frame 7) deformation which is also apparent when a crack arrives at the free end of a plexiglas specimen (see also Fig. 6, frames 22 and 23).

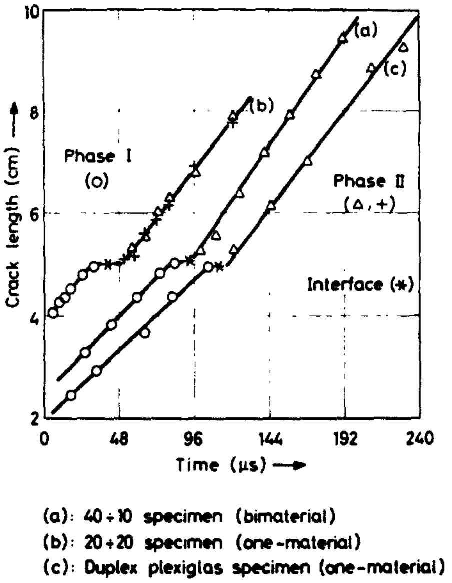


Fig. 6. Diagrams of crack length vs crack propagation time: (a) for the specimen of Fig. 3 ($v_I = 314$ m/sec, $v_{II} = 461$ m/sec); (b) for the specimen of Fig. 4 ($v_I = 354$ m/sec, $v_{IIa} = v_{IIb} = 395$ m/sec); (c) for a duplex plexiglas specimen ($v_I = 294$ m/sec, $v_{II} = 387$ m/sec).

This larger deformation of the interface and the local debonding of it at the vicinity of the crack tip, due to the reduced strength of the interface bond, resulted in considerably larger deformations of the caustic during crack arrest.

The reduced strength of the interface bond resulted in seven out of the ten experiments performed on duplex plexiglas specimens in a disruption of the interface, when the crack arrived at it, a result which appears only rarely during fracture of bimaterial epoxy resin specimens.

Figures 6(a), (b) and (c) illustrate the dependence of the crack length on time for the three specimens described previously. The slopes of the straight lines give the respective crack propagation velocities in each phase of the duplex specimens.

In all cases the same qualitative characteristics are present: A constant crack propagation velocity in phase I, a small period of crack arrest and the establishment of a new constant crack propagation velocity in phase II. Notice that in all cases, the beginning of the new fracture process, after crack arrest, with approximately the same external load conditions as by crack initiation in phase I but with much larger initial crack lengths, resulted always in a larger value of the crack propagation velocity in phases II.

The effect of the mechanical properties of the different materials used in the case of the bimaterial specimens, simply superimposes with the above described intrinsic characteristic of the fracture process of duplex specimens, thus reinforcing the increase of the crack propagation velocity (case of a more ductile phase I) or counteracting to it (case of a more brittle phase I).

The experimental results show that the main factor, which determines the dynamic crack propagation process through a bi-material interface, is not the abrupt change of the mechanical properties of the material of phase I to the properties of the material of phase II, but the existence of the interface itself and the corresponding to it microscopical perturbed interphase region, which corresponds to the area between the two phases and contains areas of adsorption intercation on both sides of the interface, as well as an area of mechanical imperfections, microcracks impurities, etc. This region has different mechanical properties than both actual phases and, when examined microscopically constitutes an effective interphase between the two existing phases (for the characteristics of the interphase region see [16]).

The mechanical properties of this microscopic interphase region do not resemble the ones of an intermediate adhesive lager. This microscopic region constitutes a discontinuous region between the actual phases of the specimen. As a definite material discontinuity, the interface plays the rote of a barrier to the crack propagation process. Similar experiments performed with other types of definite material discontinuities, such as small cracks transversely oriented to the path of the propagating crack, or small holes[17] have shown, in agreement with the conclusions of this paper, that as the crack tip arrives at the area of discontinuity a rapid change in energy forms takes place which forces the crack to arrest. The kinetic and strain energies deposited at the crack tip are first converted in energy of deformation of the boundaries of the discontinuity, thus resulting in a crack arrest (and disappearance of the caustic). This process is what we call the first period of crack arrest. As this first period of crack arrest comes to an end a delay phenomenon appears that, although the external load is still acting on the specimen, it does not create any singular stress concentration at the tip of the crack. However, in a few microseconds during the second period of crack arrest, a new singular stress concentration is created, this time "beyond the barrier" (that is in phase II of the specimen), which causes crack reinitiation.

The process of crack reinitiation is governed by the material properties of phase II, the crack length, the external load, etc.

The role of the interface on dynamic crack propagation will be better illustrated by studying the details of behavior of the stress intensity factor during the fracture process.

The first phenomenon to notice is that for all bimaterial specimens the stress intensity factor remains almost constant after the crack initiation in phase I and within a certain distance from the interface. As the crack reaches a point lying at a distance of approx. 0.2 of the width of phase I from the interface the stress intensity factor starts increasing and reaches at the interface a value of 30–50% higher than its constant value far away from the interface.

Similarly, and in all bimaterial plates tested, after the crack arrest period the crack starts propagating in phase II with a relatively low value of the stress intensity factor, to reach, later on, a considerably larger stable value. The same phenomena were also apparent during the fracture of one-material duplex specimens.

This increase of the K_I-value is a result of the interaction of the moving crack with the interface. Although the method of caustics, as all other experimental methods for the determination of the stress intensity factor, was developed in relation with the near tip solution of the singular stress field for a crack in an infinite medium, it has been shown that it can be used also in cases of cracks lying close to boundaries, other cracks, etc. since, as in the case studied of a crack approaching the interface, only the size of the caustic changes, while the shape and form of the caustic remain unaffected. Thus, while the diameters of the caustic yield the order of singularity at the crack tip the change of its size indicates and measures the influence of other discontinuities of the body lying close to the crack tip.

It seems that, as the crack approaches the interface, the relation between K_I and $\dot{a}$, cannot be considered as linear. (Constant crack propagation velocity $\dot{a}$, increase of K_I). This experimental result is common for nearly all investigations dealing with problems of crack propagation not far away from boundaries.

For example Dally *et al.*[18] investigated the crack propagation in duplex specimens with their two phases cemented together with a tough adhesive layer and concluded that the K_I-value falls rapidly as the crack approaches the interface, while the $\dot{a}$-value remains constant. Sereda *et al.*[19] studied a similar problem and stated that the K_I-value can increase or decrease, depending on the material properties of the phases, as the crack approaches an interface, without any significant change in the $\dot{a}$-value.

On the other hand, Theocaris and Andrianopoulos[20] investigated the behavior of K_I and $\dot{a}$ during fracture of PMMA specimens in dynamic three point bending and concluded that the changes in the above magnitudes, caused by the interaction of the moving crack with the neutral axis of the bent bar and its boundaries, take place in such a way that no linear relation between K_I and $\dot{a}$ can be established. Similar results can be also derived from the investigation of Kobayashi *et al.*[21] on dynamic fracture by bending.

Figure 7 illustrates the crack propagation in a duplex plexiglas specimen. The size of the caustic increases as the crack approaches the interface before crack arrest and increases also after crack reinitiation in phase II up to a final stable value. It is worthwhile noticing the significant "*outward*" deformation of the free boundary of the specimen as the crack approaches this boundary (frame 23) as well as the immediately after fracture "*inward*" deformation of the boundary (frame 24) and the subsequent creation of the caustic which travels back along the lips of the crack together with the reflected apparent wave front (frame 24),

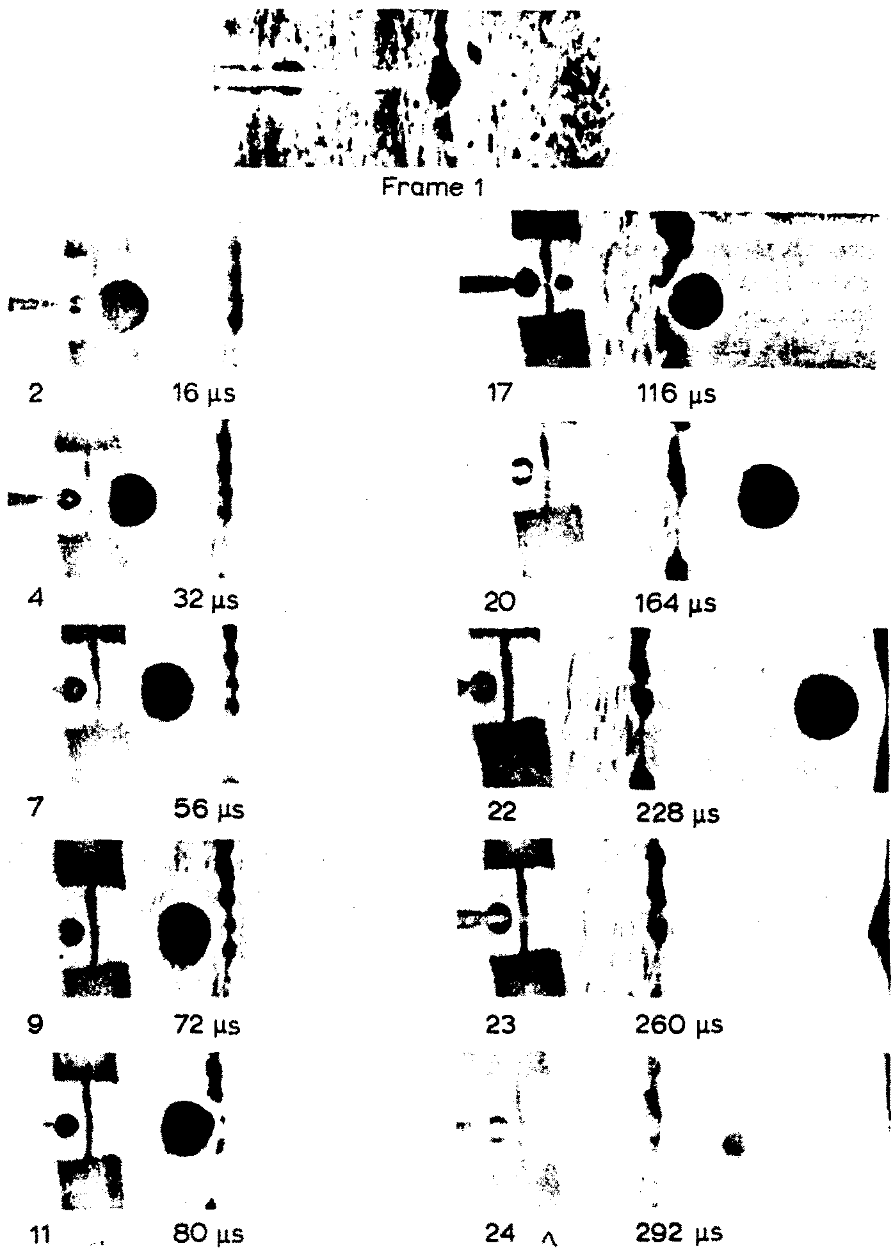


Fig. 7. Series of photographs showing crack propagation in duplex plexiglas specimens. Notice the variations in the size of the caustics. Frame 1 shows the stress distribution at the tip of a crack lying in phase I of a bimaterial specimen (30% plasticized), at a distance of 0.007 m from the interface (Phase II, 10% plasticized).

although complete failure of the plate is already accomplished (great deformation ability of the plexiglas material which is also apparent when the crack reaches the interface of the duplex plexiglas specimen). The velocity of this caustic is of the order of the velocity of the transversal (shear) waves in plexiglas which yields a proof that the travelling wave is in fact a shear wave (see also [22]).

Figure 7 presents the variation of the stress intensity factor at each phase of the duplex plexiglas specimens, normalized to its initial value at the initiation of the crack at each phase, as a function of the crack length. What is, though, most important in detecting the role of the interface in the fracture process of a duplex specimen is to investigate the behavior of the stress intensity factor at the crack tip during the period of the crack arrest.

The process of crack arrest is at first connected with the submergence of the caustic in the interface, while the interface is at the same time deformed by being "pushed" forward to the

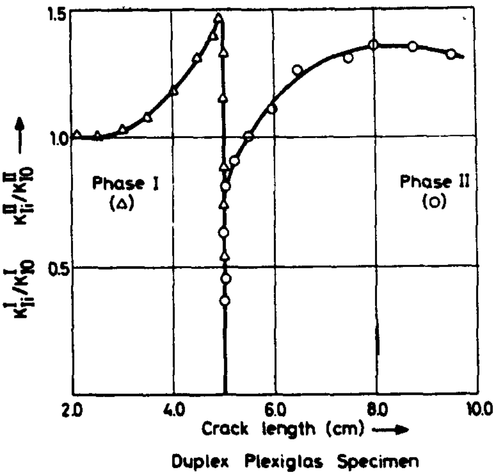


Fig. 8. Diagram of crack length vs crack propagation time for a duplex plexiglas specimen.

side of phase II. It seems that the energy initially deposited in the singular stress field at the crack tip is to a certain extent converted into deformation energy of the interface. On the other hand, and for all types of specimens, phase II does not seem to "*feel*" the singular stress distribution at the crack tip during the first period of crack arrest. Later on, the caustic emerges in phase II without being extended in phase I of the specimen.

This experimental evidence illustrates the significant difference between the dynamic and static cases of the stress distribution at the tip of a crack meeting at right angles or lying in a short distance from a rigid interface. As shown earlier[23] the solution of the static problem only considers the material characteristics of the specimens bonded together along their common interface, while the interface does not influence the static stress distribution.

The significant difference between the static and the dynamic cases is illustrated in Fig. 6 (frame I) which shows the stress distribution at the tip of a static crack, normal to the interface, in a bimaterial specimen (phase I 30% plasticized, phase II 10% plasticized epoxy polymer). The crack does not reach the interface, lying at a distance of 0.0007 m behind it in phase I. It is clear from this figure that the caustic and subsequently the singular stress field at the crack tip, created by the tensile load, are also extended in phase II of the specimen.

On the other hand, it was previously investigated[24], by the method of reflected caustics, the static problem of a crack meeting at right angles (on any other angle) the interface of two dissimilar media. The equations of the caustic were derived for this problem by putting the complex stress functions of Muskhelishvili's formulation of the plane stress problem in their asymptotic forms. By comparing the theoretically defined caustics to those obtained by the experiments, the order of the elastic stress singularity was calculated. A unique expression combining the geometrical characteistics of the caustic and the value of the stress intensity factor was established. The form of caustic calculated for this configuration corresponds only to the form of caustic created in the dynamic case during the last instant of crack arrest, just before crack reinitiation, when the final, semicircular, crack—arrest caustic—form has emerged from the interface.

The above results clearly indicate that it is not possible to investigate the process of crack propagation through a rigid interface by means of the static analysis, where only the influence of the different mechanical properties of the materials used is involved, or to use the results of a theoretical analysis of the static case in order to "predict" the possible propagation patterns of a moving crack which meets the interface, as it has been applied previously[25].

Considering the above described experimental evidence we can formulate the following rule for the experimental evaluation of the stress intensity factor during the crack arrest process.

The first period of crack arrest, that is when the previously in phase I running caustic submerges in the interface seems to be depending only on the presence of the interface. We assume therefore that during this period the singularity at the crack tip retains its previous value of $p = 0.5$. The caustic formed during the crack arrest period represents arcs of the previous "entire" caustic, which is being now progressively submerged in the interface. We assumed further that these caustic-forms corresponded to entire caustics with a transverse diameter of $D_t = 2l/\pi$, where l is the length of the arc of the respective caustic form, and we calculated the stress intensity factor accordingly.

The second period of carck arrest is characterized by the reestablishment of the stress field behind the interface—the "barrier" has been "overleaped"—and ends up with a caustic form similar to that of the static case.

Thus, during this period, a new order of singularity, different from $p = 1/2$, is established. This singularity is not complex only in the case when the crack meets the interface at right angles[26] and for our cases of bimaterial specimens deviates only insignificantly from the value of $p = 0.5$[24]. The extreme cases were: For the specimens with phase I an unplasticized epoxy polymer and phase II a 40% plasticized one: $p = 0.56$, for the inverse case where phase I contained a 40% plasticized epoxy and phase II was unplasticized we had $p = 0.475$. Again, for this second period of crack arrest the stress intensity factor for the final caustic form was calculated according to Ref. [24]. The final caustic form corresponded with great accuracy to the static caustic at the tip of a crack meeting at right angles a bimaterial interface and was so calculated accordingly (see also [27]). At the same time we assumed that the preliminary arc forms of the emerging caustic correspond to final arrest caustic forms with a transverse diameter of $D_t = 2l/\pi$ where l is the length of the arc of the respective part of the caustic.

This rule for the calculation of K_I from preliminary caustic forms is in accordance with the

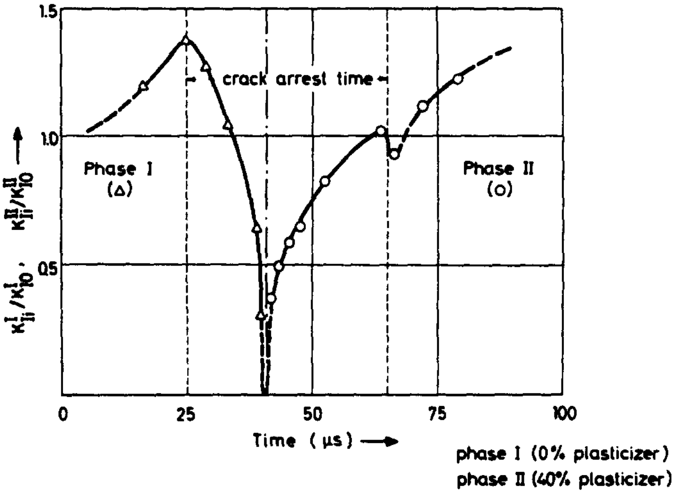


Fig. 9. Diagram showing the variation of the stress intensity factor with time during crack arrest, normalized to its initial value at the initiation of the crack at each phase, for a (0–40) bimaterial specimen.

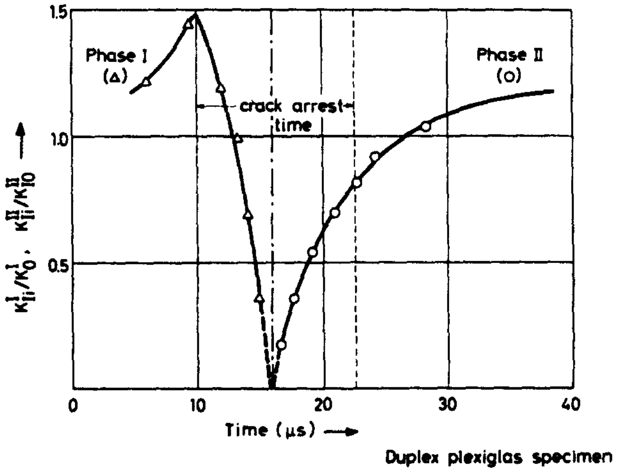


Fig. 10. Diagram showing the variation of the stress intensity factor with time during crack arrest, normalized to its initial value at the initiation of the crack at each phase, for a duplex plexiglas specimen.

experimental evidence: The stress intensity factor can be considered, with a good approximation, as a function of the half perimeter s of the caustic ($2s \simeq \pi D_t$). At the same time the boundary conditions for the initial and final instants of crack arrest ($K_I = f(D_t)$) are satisfied (see also [24]).

After crack reinitiation we assumed that the order of singularity $p = 0.5$ is again attained.

Figures 9 and 10 show the variation of the stress intentity factor with time, normalized to its initial value at the initiation of the crack at each phase, for a 0–40 bimaterial specimen ($p = 0.56$) and a plexiglas duplex specimen ($p = 0.5$).

During the first period of crack arrest the caustic (appearing only in phase I) submerges in the interface, until it completely disappears, which means that the stress intensity factor takes continuously lower values up to zero. Subsequently the caustic emerges in phase II of the specimen continuously taking larger values, up to the critical value of crack reinitiation. The assumption that the crack-tip singularity abruptly retains the value of 0.5 after crack reinitiation often means that the K_I-curve follows a bend immediately after crack-reinitiation.

The different order of singularity than the typical 0.5 singularity influences the variation of the K_I-value only insignificantly in the second period of the crack arrest process of the bimaterial specimen. It is obvious that the main changes in the value of K_I are due to the singluar stress field disappearance and reappearance at the interfacial discontinuity.

The previously mentioned investigations on fracture of duplex specimens were discussed in [1]. However, Sereda *et al.*[19] were not able to study the behavior of K_I stress intensity factor and its variation during the crack arrest period. On the other hand Dally *et al.*[18] claims a completely different behaviour of K_I during crack arrest and a simultaneous appearance of stress–strain fields at both sides of the arrested crack in the intermediate layer. According to our opinion these different results of the above authors are mainly due to the existence of the intermediate adhesive layer in their experiments. This layer was not only tougher than the materials of the main phases but was also considerably thick relatively to the zone of perturbation at the interface. In their cases the crack stopped in the cement layer having practically left behind phase I and without touching the boundary of phase II. At this position a possibility really exists for a rather symmetrical build-up of the stress strain field around the crack tip. On the other hand, the experimental method of dynamic photoelasticity used by the above authors does not allow such a close approximation to the crack tip as does the method of caustics, and cannot also avoid the influence of the (constant) part of the stress field at the boundaries of the specimen phases, which does not influence the shape and form of the caustic.

6. CONCLUSIONS

It was shown in this paper that the dynamic process of crack arrest at the interface of a bimaterial or duplex specimen depends primarily on the existence of this interface itself and, auxiliarily, on the material properties of the different phases.

The interface constitutes a definite material discontinuity which hinders the propagation of the singular stress field of the crack tip by mainly disturbing the process of propagation of the stress waves emanating from the tip of the crack. The fracture stress waves build up the main mechanism of "information" of the material lying ahead of the tip of the moving crack about its approach and so "prepare" this material for the further crack extension.

The crack arrest process consists of two distinct periods. During the first period of crack arrest the singular stress field at the tip of the arresting crack vanishes, although the external load, which caused the singularity, continues to act on the specimen. The previously deposited strain energy at the crack tip is being now absorbed by the material lying in the neighborhood of the crack tip.

The second period of crack arrest is characterized by the gradual build-up of a new singularity, (this time beyond the "barrier"), which ends by causing a reinitiation of the crack.

The experimental evidence supported the formulation of a rule for evaluating the stress intensity factor during both periods of crack arrest. Finally the relationship between crack propagation and the stress-wave emission during fracture was discussed.

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